[This question paper contains 4 printed pages.]

Your Roll No. 2022

C

Sr. No. of Question Paper : 1015

Unique Paper Code : 32221501

Name of the Paper : Quantum Mechanics and Applications

Name of the Course : B.Sc. (Honors) Physics

Semester : V

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **FIVE** questions in all. Question No. **1** is compulsory.
3. **All** questions carry equal marks.
4. Non programmable calculators are allowed.

1. Attempt any **FIVE** of the following :

(a) Calculate the commutator $[\widehat{L_x}, \widehat{p_x}]$. (given $[\hat{x}, \widehat{p_x}] = i\hbar$).

(b) The wave-function of a particle is $\psi(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$ for $0 \le x \le L$. Determine the probability of finding the particle at x = L/3 for n = 3 state.

P.T.O.

(c) Derive the relation between 'magnetic dipole moment' and 'orbital angular momentum' of an electron revolving around a nucleus.

(d) Write the quantum numbers for the state represented by $4^2F_{5/2}$.

(e) Normalize the wave function e^{-ax^2} in a one-dimensional space.

(f) A free particle of mass m is described by the wave-function $\psi(x) = A\exp(i\mu x)$ where A and μ are constants. Determine the probability current density for this particle.

(g) Determine the uncertainty in position for the normalized wave-function $\psi(x) = \left(\frac{2\alpha}{\pi}\right)^{1/4} e^{-\alpha x^2}$ for $-\infty < x < \infty$. (5×3=15)

2. (a) Explain the concept of expectation values. Give expressions for the expectation values of velocity, momentum and energy in terms of respective operators in three dimensions. Mention the difference between expectation values and eigenvalues of an operator corresponding to a dynamical variable.

(b) The wave-function of a particle of mass m is given by

$$\Psi(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} e^{-\beta^2 x^2/2} \text{ for } -\infty < x < \infty.$$

Determine the total energy of the particle, if potential energy is $V(x) = \frac{1}{2} m\omega^2 x^2$. (7.8)

3. The Gaussian wave packet for a free particle is defined by the wave function

$$\Psi(x,0) = N \, exp\left(-\frac{x^2}{2\sigma^2} + ik_0 x\right).$$

Prove that the centre of this Gaussian wave packet travels with a velocity $v = \frac{k_0 \hbar}{m}$.

(Use $\int_{-\infty}^{\infty} e^{-x^2/\sigma^2}\, dx = \sigma\sqrt{\pi}$ and $\int_{-\infty}^{\infty} e^{-(ax^2 \pm bx)}\, dx = \sqrt{\frac{\pi}{a}} e^{b^2/4a}$).

(15)

4. (a) Solve the Schrodinger equation for a linear Harmonic Oscillator and obtain first two eigenfunctions. (10)

(b) Find ΔX and ΔP for the ground state eigenfunction of linear Harmonic Oscillator and obtain the uncertainty principle. (5)

5. (a) The 'θ' equation obtained after applying separation for variables to the Schrodinger equation for a 3D hydrogen atom in spherical polar coordinates, is given by

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{d\Theta}{d\theta}\right) + \left(\lambda - \frac{m_l^2}{\sin^2\theta}\right)\Theta = 0.$$

Solve the above equation for $m_l = 0$ (or otherwise) to show that

$$\lambda = l(l+1), \quad l = 0,1,2, \dots\dots\dots$$ (12)

P.T.O.

(b) An electron in a hydrogen atom is in a state described by

$$\psi = \frac{1}{\sqrt{6}}[2\psi_{100} + \psi_{211} + \psi_{21-1}]$$

Calculate the expectation value of $\hat{L}_z$ in this state.

(Given $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ and

$$\psi_{100} = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} e^{-r/a_0}$$

$$\psi_{211} = \frac{1}{8\sqrt{\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0} \sin\theta\, e^{+i\phi})$$ (3)

6. (a) What is Larmor Precession? Draw the relevant diagram and derive the expression for Larmor frequency.

(b) A beam of silver atoms moving with a velocity 10^7 cm/s passes through a magnetic field of gradient 0.5 Wb/m²/cm for 10 cm. What is the separation between the two components of the beam as it comes out of the magnetic field? (8,7)

7. (a) What is spin orbit coupling? Explain the fine structure splitting in the energy levels due to this. For the 2p level of the hydrogen atom with $E_n = -3.14$ eV, evaluate the fine structure splitting.

(b) Consider a two-electron system with $l_1 = 1$, $l_2 = 1$. Explain the LS coupling scheme in such a case. Write the spectral notation for each state. (10,5)

[This question paper contains 4 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 1051 **C**

Unique Paper Code : 32221502

Name of the Paper : Solid State Physics

Name of the Course : **B.Sc. (Hons.) Physics CBCS (Core)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions in all.
3. Question number **1** is compulsory.
4. **All** questions carry equal marks.

1. Attempt any **five** of the following : (5×3=15)

   (a) Prove that for a SC lattice, $d_{100} : d_{110} : d_{111} = \sqrt{6} : \sqrt{3} : \sqrt{2}$; where 'd' represents interplanar distance in a crystal.

P.T.O.

(b) An element has a cubic structure having lattice constant as 4.28 Å, and with two of its atoms in the unit cube at (0,0,0) and (1/2, 1/2, 1/2). Find out the distance between nearest neighbours in this element.

(c) The Debye Temperature for Diamond is 2230 K. Calculate the highest possible vibrational frequency.

(d) The energy near the top of the valence band of a crystal is given by $E = -Ak^2$, where $A = 10^{-39}$ Jm^2 and k is the wave vector. An electron with wave vector $k = 10^{10}\, \hat{k}_x\ m^{-1}$ is removed from an orbital in a completely filled valence band. Find the effective mass, momentum and energy of the hole. Given Planck's constant $h = 6.62 \times 10^{-34}$Js.

(e) What are the basic assumptions of Drude's model for describing electron motion in metals.

(f) Distinguish between dia-, para- and ferromagnetism.

(g) Calculate the electronic polarizability of Neon. The radius of Neon atom is 0.158 nm. ($\varepsilon_0 = 8.854 \times 10^{-12}$ F/m).

(h) Calculate the critical current which can flow through a long thin superconducting wire of aluminium of diameter 10^{-3} m. The critical field for aluminium is 7.9×10^3 A/m.

2. (a) What is Geometrical structure factor? Derive its expression for FCC structure having identical atoms. Will reflection from (211) plane be possible for this structure? (10)

(b) Show that reciprocal lattice of a BCC lattice is a FCC structure. (5)

3. (a) Derive the expression for specific heat of a solid based on Einstein's model. Explain why this model was not successful. (8)

(b) Deduce the dispersion relation for a linear monatomic chain of atoms and show that the group and phase velocities of a wave are same in the long wavelength limit. (7)

4. (a) Find the expression for the Hall coefficient of a semiconductor in which both electrons and holes are present in equal concentrations. How will this expression change if the hole concentration is twice the electron concentration and vice-versa? Also, explain how will this expression be modified if the semiconductor is heavily doped with p-type impurity or n-type impurity? (12)

(b) Distinguish between direct and indirect band gap with the help of diagram. (3)

5. (a) Derive the expression for Curie-Weiss law using Weiss theory of Ferromagnetism. (6)

(b) Discuss the concept of hysteresis and show that the B-H hysteresis loop gives the value of energy dissipated per cubic meter of the material per cycle of magnetization. (6)

(c) A magnetic substance has 10^{28} atoms/m^3. The magnetic moment of each atom is 1.8×10^{-23} Am^2. Calculate the paramagnetic susceptibility at 300K. What would be the dipole moment of a bar of this material 0.1 m long and having cross-sectional area of 1 cm^2 in a field of 8×10^4Am, $\mu_0 = 4\pi \times 10^{-7}$ henry/m, $k_B = 1.38 \times 10^{-23}$ J/K. (3)

6. (a) Explain the concept of Local Electric Field in a dielectric and derive its expression for structures possessing cubic symmetry. (8)

(b) Obtain Clausius-Mossotti's relation between polarizability and dielectric constant of a solid. (7)

7. (a) Explain the phenomenon of superconductivity. Derive London's first and second equations and discuss penetration depth in a superconductor with the help of a diagram. (12)

(b) Show that the susceptibility of superconductors is −1 and relative permeability is zero. (3)

[This question paper contains 6 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 1140 **C**

Unique Paper Code : 32227502

Name of the Paper : Advanced Mathematical Physics (DSE – Paper)

Name of the Course : **B.Sc. (Hons) Physics (CBCS – LOCF)**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **five** questions in all taking at least **two** questions from each section.

3. **All** questions carry equal marks.

SECTION A

1. (a) Is the set $\{1, -1, i, -i\}$ a group under multiplication? (5)

P.T.O.

(b) Show that W is not a subspace of vector space V where

$$W = \{f : f(7) = 2 + f(1)\}.$$ [5]

(c) Consider the following subspace of R^4:

$$W = \{(a, b, c, d) : b + c + d = 0\}$$

Find the dimension and basis of W. (5)

2. (a) Determine whether the transformation, $T: R^3 \rightarrow R^3$ defined by,

$$T(x, y, z) = (x + 2y - 3z, x + y + z, 7x - y + 5z)$$

is linear or not. (5)

(b) Let $T: R^3 \rightarrow R^3$ be defined by

$$T(x, y, z) = (x + y - 2z, x + 2y + z, 2x + 2y - 3z).$$

Show that T is a non-singular transformation. (5)

(c) Linear transformation T on R^2 is defined as

$$T(x, y) = (3x - 4y, x + 5y)$$

Find the matrix representation of T relative to the u-basis: $\{u_1 = (1, 3)$ and $u_2 = (2, 5)\}$. (5)

3. (a) Assume that A, $I - A$, $I - A^{-1}$ are all non-singular matrices, show that :

$$(I - A)^{-1} + (I - A^{-1})^{-1} = I \qquad (5)$$

(b) Find the condition for the following matrix to be orthogonal

$$\begin{bmatrix} a+b & b-a \\ a-b & a+b \end{bmatrix}. \qquad (5)$$

(c) Evaluate C^{20}, where $C = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$. (5)

4. (a) Given a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, prove that its eigenvalue equation is given by

$\lambda^2 - \lambda \mathrm{Tr}(A) + \det(A) = 0.$ (5)

(b) Solve the following system of differential equations using matrix method

$$\dot{y} = z$$
$$\dot{z} = y$$

where, $y(0) = 4$, $z(0) = 2$. (10)

SECTION B

5. (a) Prove that $\delta_{pr} \in_{prs} = 0$. (3)

(b) If $B_{ps} = \in_{psk} A_k$, show that

$$A_u = \frac{1}{2} \in_{ups} B_{ps}$$ (5)

(c) If a tensor A_{ijklm} is symmetric with respect to two indices i and k in the coordinate system x_i, then show that it is symmetric with respect to the same indices in any other co-ordinate system $\bar{x}_p$. (7)

6. (a) Prove that

$$\in_{abc} \in_{pkm} = \begin{vmatrix} \delta_{ap} & \delta_{ak} & \delta_{am} \\ \delta_{bp} & \delta_{bk} & \delta_{bm} \\ \delta_{cp} & \delta_{ck} & \delta_{cm} \end{vmatrix}$$

and hence show that

$$\in_{ibc} \in_{ikm} = \delta_{bk}\delta_{cm} - \delta_{bm}\delta_{ck}$$ (8)

(b) Using tensor methods, verify the identity

$$\vec{\nabla}(\vec{A}.\vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A}.\vec{\nabla})\vec{B} + (\vec{B}.\vec{\nabla})\vec{A}$$

(7)

7. (a) Stress tensor (p_{ij}) satisfies the equations $p_{ij} \in_{ijk} = 0$ and $p_{ij} = f_i n_j$, where f_k is the restoring force per unit area along x_k - axis and $\hat{n}$ is the arbitrary unit vector. Prove that stress tensor is a symmetric tensor of order two. (5)

(b) Stress tensor and strain tensor are related as

$$p_{ij} = \omega_{ijks} e_{ks},$$

where, elastic tensor ω_{ijks} is symmetric in i, j and k, s and its general form is

$$\omega_{ijks} = \lambda \delta_{ij} \delta_{ks} + \mu \delta_{ik} \delta_{js} + \gamma \delta_{is} \delta_{jk}.$$

Prove that

(i) $\omega_{ijks} = \lambda \delta_{ij} \delta_{ks} + \mu(\delta_{ik} \delta_{js} + \delta_{is} \delta_{jk})$

(ii) $p_{ii} = (3\lambda + 2\mu) e_{ii}$ (7)

(c) Let the state of stress at a point in a solid body is given by

$$S_{ik} = \begin{bmatrix} 10 & 10 & 20 \\ 10 & 20 & 0 \\ 20 & 0 & 55 \end{bmatrix}$$

Find the normal stress and shear stress on the surface defined by

$$3x - 2y + 2z = 10 \quad (3)$$

8. (a) Prove that g_{ij} is a covariant tensor of rank 2. (5)

(b) If $ds^2 = 3(dx^1)^2 + 5(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4\,dx^2dx^3$

Find the matrices

(i) g_{ij} (4)

(ii) g^{ij} (4)

(iii) the product of (g_{ij}) and (g^{ij}) (2)

[This question paper contains 8 printed pages.]

Your Roll No. 2022

Sr. No. of Question Paper : 1144 **C**

Unique Paper Code : 32227506

Name of the Paper : Astronomy & Astrophysics

Name of the Course : **B.Sc. (Hons) Physics – DSE**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question No. **1** which is compulsory.

3. Attempt any **four** questions from the remaining.

1. Attempt any **four** of the following : (5×3=15)

   (a) The lunar parallax determination was done by Hipparchus (200-BC). The measurement was done during a total solar eclipse at Syene and a partial eclipse at Alexandria where about 1/5~th of the

30~arcmin Sun's disk was visible. If the latitudes of Syene and Alexandria are 41° and 31° respectively, find the Earth- Moon distance in terms of radius of Earth.

(b) Make a rough estimate of Earth's mean temperature T by assuming that it is a perfect blackbody and spherically symmetric. Assume that it absorbs the entire solar radiation incident upon it and radiates as a blackbody at temperature T.

(c) What are circumpolar stars? What would be the declination of circumpolar stars for an observer at latitude 30°N?

(d) The apparent magnitude of Full Moon is −12.73 and that of Jupiter is −2.60. Calculate their brightness ratio.

(e) Determine the magnitude limit in the visible range that can be detected by a 1.04 m telescope.

(f) Give two arguments in support of the expanding universe as compared to the steady state universe.

(g) At what wavelength does a star with the surface temperature of 4000° K emit most intensely?

2. (a) Define the essential elements of the Local Equatorial Coordinate System and show them on an appropriate diagram. Draw the diurnal motion of a star in this coordinate system. Also draw the diurnal motion of Sun on Solstices at latitude 23°. (9)

(b) The image of an astronomical object forms at the focus of a telescope. For astronomy, one is interested in its 'angular size' (measured in arcseconds) but to photograph it you are interested in how big it will be compared to the size of your film or digital sensor array (measured in mm). An astronomer wants to design a camera so that each pixel views an angle of only 0.5 arcsec. If the width of each pixel is 8~micron, what is the focal length needed for the telescope? If a digital camera array measures 20 mm across and consists of 2048 pixels, what will the telescope focal length have to be so that the array can be used to photograph a star cluster with a diameter of 15 arcmin? (6)

3. (a) Write the Saha Ionization equation. What is its significance? Draw a schematic HR diagram. (9)

(b) Calculate the frequency shift produced by the normal Zeeman effect in the center of a sunspot that has a magnetic field strength of 0.3~T. By what fraction would the wavelength of one component of the 630.25 nm Fe I spectral line change as a consequence of a magnetic field? (6)

4. (a) Explain the meaning of scale factor a(t) as defined in cosmology. Starting from Newtonian cosmology derive Friedmann equation in terms of scale factor and explain the model(s) of the universe based on derived equation. Using the Friedmann equation, show that for a flat universe having single fluid as non-relativistic matter or dust, the scale factor evolves as $a(t) \alpha\, t^{2/3}$. (9) 6.

(b) For an observer at latitude 42.5°N and longitude 71°W, estimate the local time of sun rise on 21 December. If the observer's civil time is –5 hrs from GMT find the time of sunrise as per observer's standard watch. Ignore refraction of the atmosphere, the size of the solar disc.

(6)

5. (a) State and derive Viriai theorem for bound and stable system of N particles under mutual gravitational attraction. How it can be used to make an order-of-magnitude estimate of the average temperature in the interior of the star.

(9)

(c) On 9 March 2011 the Voyager probe was 116.406 AU from the Sun and moving at 17.062 km/s. Determine the type of orbit the probe is on? What is the apparent magnitude of the Sun as seen from Voyager?

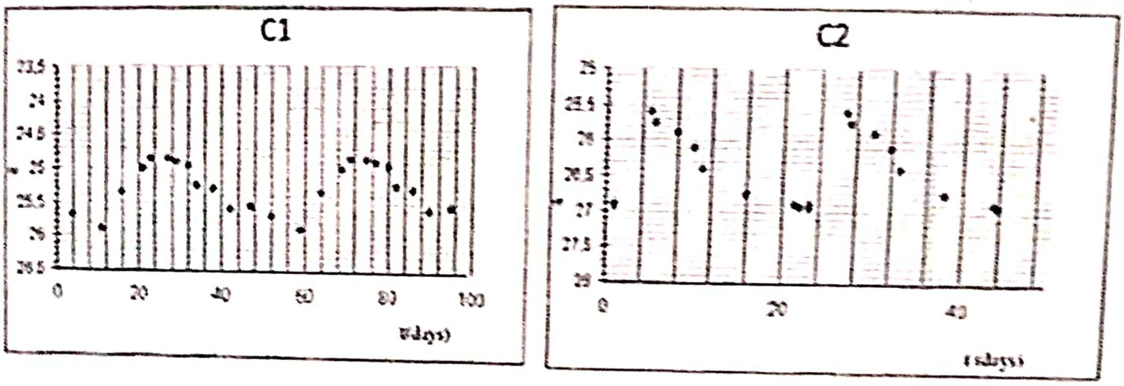
(6)

6. (a) What is the meaning of differential rotational of Milky way galaxy? Obtain the expression for the Oort's constants and discuss their significance.

(9)

(b) Cepheids are very bright variable stars whose mean absolute magnitudes & are functions of their pulsation periods given by $M = -1.21 - 2.88 \log_{10} P$.

Two panels show the light curves of two Cepheids.



Estimate the distances to each of these two Cepheids. Roughly estimate the uncertainty in the distance determination. Comparing the difference between the distances of the two stars with the typical size of a galaxy, would it be likely for these two stars to be in the same galaxy? (6)

7. (a) The coordinates transformation of a point P between two rectangular coordinate frames O_{xyz} and $O'_{x'y'z'}$ such that the O′ frame is obtained from the O frame by rotating it around the x axis by an angle χ is given by

$$\cos\psi'\cos\theta' = \cos\psi\cos\theta$$

$$\sin\psi'\cos\theta' = \sin\psi\cos\theta\cos\chi + \sin\theta\sin\chi$$

$$\sin\theta' = -\sin\psi\cos\theta\sin\chi + \sin\theta\cos\chi$$

The angle $\psi(\psi')$ are measured counterclockwise from the positive $x(x')$ axis along the $xy(x'y')$ plane. The angle $\theta(\theta')$ is the angular distance from the xy $(x'y')$ plane. Derive triangulation formulas for the spherical triangle ABC

$$\sin B \sin a = \sin A \sin b$$

$$\cos B \sin a = \cos A \sin b \cos c + \cos b \sin c$$

$$\cos a = -\cos A \sin b \sin c + \cos b \cos c$$

by expressing the spherical coordinates ψ, θ, ψ' and θ' in terms of the appropriate sides a,b,c and angles A,B,C of the triangle. From this deduce the sine formula. (9)

(b) The Sun's "surface" is a thin layer of the solar atmosphere called the photosphere. The characteristic temperature of the photosphere is Te = 5777~K, and it has about 500000 hydrogen atoms for each calcium atom with an electron pressure of about 1. 5 Nm^{-2}. Estimate (i) ratio of ionized to neutral hydrogen and (ii) ratio of hydrogen atoms are in the first excited state to neutral hydrogen. Find the relative number of hydrogen atoms capable of producing Balmer absorption lines. The energy needed to convert it from H I to H II, is χ_I =13.6eV. (6)

Constants

Universal Gravitational Consntant $G = 6.67 \times 10^{-11} m^3 kg^{-1} m^{-2}$

Boltzmann Constant $k_B = 1.38 \times 10^{-23}\, JK^{-1}$ Stefans Constant $\sigma = 5.67 \times 10^{-8}\, Wkm^{-2}K^{-4}$

Parsec $1\, pc = 3.1 \times 10^{16}\, m$ Astronomical Unit $1\, AU = 1.50 \times 10^{11}\, m$

Mass of Earth $M_E = 5.97 \times 10^{24}\, Kg$ Radius of Earth $R_E = 6.36 \times 10^{6}\, m$

Mass of Sun $M_{\odot} = 1.99 \times 10^{30}\, Kg$ Radius of Sun $R_{\odot} = 6.96 \times 10^{8}\, m$

Apparent Magnitude of the Sun $m_{\odot} = -26.72$ Solar constant $S = 1370\, Wm^{-2}$

Density of the solar photosphere $\rho_{photosphere} = 4.9 \times 10^{-6}\, kgm^{-3}$

Mass of He^3 nucleus $M_{He^3} = 2808.30\, MeV$ Mass of He^4 nucleus $M_{He^4} = 3727.40\, MeV$

Mass of He nucleus $M_H = 938.27\, MeV$ Naked Eye limit of Apparent Magnitude $m_e = 6$

Oort constants $A = 15\, kms^{-1}kpc^{-1}$ & $B = -10\, kms^{-1}kpc^{-1}$

Velocity and distance of Sun about Galactic Center $v_{\odot} = 218 kms^{-1}$ & $r_{\odot} = 8\, kpc$

Mass of electron $m_e = 9.1 \times 10^{-31}\, kg$ & Charge of electron $e = 1.6 \times 10^{-19}\, C$